Characterization of Rectangular Waveguide With a Pseudochiral Ω Slab

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***Abstract*—A mode matching approach has been employed for evaluating the matrix $\underline{\underline{S}}$ of the section of rectangular waveguide with pseudochiral Ω slab. Scattering characteristics for different localizations of the Ω slab in the guide are predicted numerically and verified by measurements. Based on the numerical analysis, the scattering properties of the guides are discussed. Scattering parameters are next applied in an optimization procedure to extract constitutive parameters of the Ω slab used in experiment.**

***Index Terms*—Pseudochiral media, scattering matrix, waveguides.**

I. Introduction

Electromagnetic properties in Ω guides have been recently analyzed by several authors [1]–[4]. Engheta and Saadaun [1], [2] have studied modes in rectangular guide with Ω slab shown in Fig. 1. They have predicted the field displacement effect similar to the one appearing in ferrite waveguides. It was noted [4] that the field displacement is always perpendicular to the plane of magnetic and electric field vectors induced in the Ω particles. Hence, this effect was also perceived [4] in the guides where the Ω slab takes on other positions. The novel properties of Ω guides have suggested their possible application in the design of microwave components [1]–[5] so the determination of Ω material parameters [5] becomes an important task. For this reason, we have decided to investigate the scattering problem of the Ω waveguide shown in Fig. 1, which has no analysis yet in literature. The analysis and simulation of the scattering characteristics are carried out by using the full-wave theory based on the mode-matching method. The Ω slab is assumed to be homogeneous and defined by constitutive equations [1]. The slab is extended over the waveguide height and can be arbitrarily placed in the guide. The excitation is chosen to be TE_{no} modes of the input rectangular waveguides so that the scattered fields, due to the localization of the Ω particles in the slab [1], can be defined only by TE_{no} waves. In our approach, the influence of the evanescent higher-order modes that exists near the waveguide junction interfaces is considered and, therefore, it yields very reliable and accurate values for scattering parameters. The calculated and measured results were used to extract three complex Ω material parameters. This approach is based on the Marquardt–Levenberg optimization procedure.

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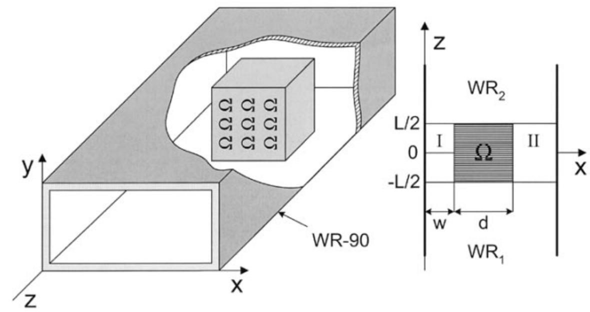


Fig. 1. Rectangular waveguide with Ω slab and geometry of the investigated structure.

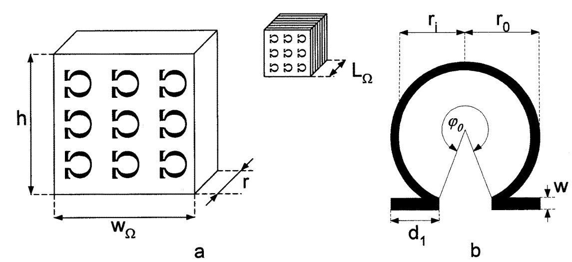


Fig. 2. (a) An Ω sample used in experiment, parameters $W_\Omega = 10$ mm, $L = 10$ mm, $h = 10.2$ mm, and $l_\Omega = 0.2$ mm; (b) a single Ω element, parameters $r_i = 0.915$ mm, $r_o = 1.046$ mm, $w = 0.131$ mm, $d = 2 * d_1 = 1.6$ mm, and $\varphi_0 = 5.96$ rad.

II. Rectangular Waveguide With Ω Slab

A. Specification of the Ω Slab

The Ω particles are arranged in the slab as shown in Fig. 2. The medium parameters appearing in the constitutive equations have dyadic representation. The relative electric permittivity and magnetic permeability are of the form $\overline{\varepsilon} = \varepsilon(\vec{i}_x\vec{i}_x + \vec{i}_z\vec{i}_z) + \varepsilon_y\vec{i}_y\vec{i}_y$ and $\overline{\mu_c} = \mu(\vec{i}_x\vec{i}_x + \vec{i}_y\vec{i}_y) + \mu_z\vec{i}_z\vec{i}_z$. The magnetoelectric coupling is defined by dyadics $\overline{\Omega}_{yz} = \pm\Omega\vec{i}_y\vec{i}_z$ and $\overline{\Omega}_{zy} = \mp\Omega\vec{i}_z\vec{i}_y$ where Ω is a pseudochirality admittance. The sign of Ω changes when the Ω particles take on opposite positions. The sample of Ω used in our experiments was fabricated as compound of 30 duroid plates ($\varepsilon = 2.2$) with a chemically etched matrix of nine Ω particles, as illustrated in Fig. 2(a). The polystyrene glue used to stick together the Ω plates has $\varepsilon \sim 2.4$. In order to calculate the unknown material parameters, we applied the equivalent model of Ω medium presented by Engheta and Saadaun in [2]. Frequency responses are plotted in Fig. 5 in accordance with experiment for bandwidth 8–10 GHz.

B. Symmetry Properties of the Ω Waveguide

Earlier, Engheta and Saadaun indicated [1], [2] the different modal properties of the guides with opposite arrangements of

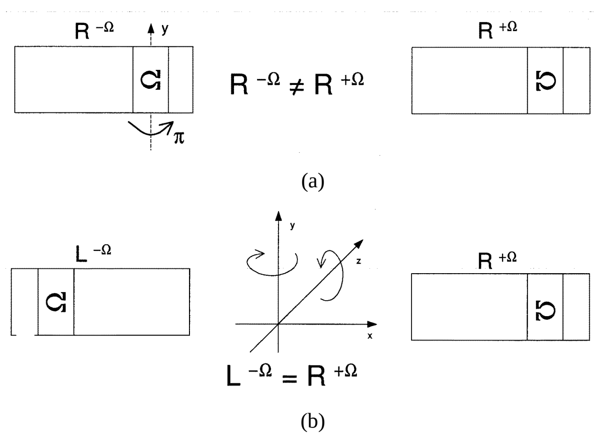


Fig. 3. Symmetry properties of the Ω guide: (a) an Ω slab rotates of π; (b) an Ω waveguide rotates of π.

Ω particles in the slab. It occurs when Ω particles are turned of π with respect of their local y or z axis. After transformation [see Fig. 3(a)], the $R^{+\Omega}$ changes to the $R^{-\Omega}$ guide, having different wave properties. Another situation is shown in Fig. 3(b), where after guide rotation with respect to the y or z axis or to the reflection in the plane perpendicular to the x axis, the $R^{+\Omega}$ is transformed to $L^{-\Omega}$. Here, both of the guides preserve invariant wave properties. In general, for investigated guides, one can develop the following symmetry condition $R^{+\Omega} = L^{-\Omega}$. In the situation when the slab in the guide R or L is turn of π their modal properties are different, so $R^{+\Omega} \neq L^{+\Omega}$. The above discussed properties indicate the appearance of the field displacement effect in the investigated guide. There is much resemblance between the pseudochiral and the ferrite waveguides. In reality, if pseudochiral and ferrite waveguides are compared, the similar changes in field displacements are observed in both guides when the Ω slab is turned of π and the magnetization of the ferrite is reversed. However, the Ω guide has the same properties regardless of the propagation direction. Hence, it is a reciprocal guide in contrary to the ferrite guide, which is a nonreciprocal structure.

C. Scattering Matrix

Fig. 1 shows the geometry of the investigated waveguide and the location of the coordinate system used. The structure consists of two waveguide regions ($WR1$, $WR2$) and a Ω interaction region. In the waveguide regions, a set of TE_{no} modes is to be considered. The modal fields in the Ω region are formulated by the transfer matrix approach [6]. Then, the continuity of the transverse electric and magnetic fields over each interface $z = \pm L/2$ yields, respectively

$$\begin{aligned}\sum_n (a_{1n} + b_{1n})\vec{e}_n &= \sum_n \left(A_n + B_n e^{-\gamma_{\Omega n} L}\right) \vec{e}_{\Omega n}\\ \sum_n (a_{1n} - b_{1n})\vec{h}_n &= \sum_n \left(A_n - B_n e^{-\gamma_{\Omega n} L}\right) \vec{h}_{\Omega n}\\ \sum_n (a_{2n} + b_{2n})\vec{e}_n &= \sum_n \left(A_n e^{-\gamma_{\Omega n} L} + B_n\right) \vec{e}_{\Omega n}\\ \sum_n (-a_{2n} + b_{2n})\vec{h}_n &= \sum_n \left(A_n e^{-\gamma_{\Omega n} L} - B_n\right) \vec{h}_{\Omega n} \qquad (1)\end{aligned}$$

where a_{in} and b_{in}, $i = 1, 2$ are complex coefficients of the incident and reflected waves and $\vec{e}_n, \vec{h}_n$ are the transverse electric and magnetic field functions in $WR_{1,2}$. A_n and B_n are the forward and backward wave complex coefficients,$\gamma_{\Omega n}$ is the propagation constant, and $\vec{e}_{\Omega n}, \vec{h}_{\Omega n}$ are the transverse eigenfields in the Ω region. Equation (1) is converted into a linear matrix form taking the inner products of both sides of this equation with eigenfields of orthonormal sets of modes in WR guides. In this approach, the Ω region fields are orthogonalized by eigenfields of WR guides. The resulting matrices are truncated to a total of N modes in each of the region. After some manipulations, the amplitudes of the fields in the Ω_R region are eliminated. Finally, the solutions is expressed in the scattering matrix formulation as

$$\begin{bmatrix}\underline{b}_1\\ \underline{b}_2\end{bmatrix} = \begin{bmatrix}\underline{\underline{S}}_{11} & \underline{\underline{S}}_{12}\\ \underline{\underline{S}}_{21} & \underline{\underline{S}}_{22}\end{bmatrix} \begin{bmatrix}\underline{a}_1\\ \underline{a}_2\end{bmatrix}. \qquad (2)$$

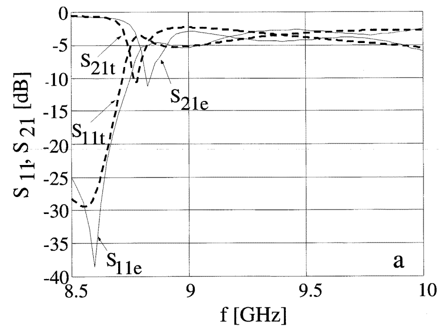
III. VERIFICATION AND RESULTS

The convergence behavior of the modal expansion method used was investigated by studying the magnitude of the S_{11} and S_{21} for the different slab positions of w as a function of the number N of eigenfields considered in (1). We have found that good convergence is already protected by few modes. However, the precise computation of higher-order mode coefficients is required for accurate field patterns, so N 12 eigenfunctions have to be taken into account. First, we verified the predicted guide behavior for two symmetrical position of the Ω slab in the guide. Calculated and measured frequency responses of the reflection S_{11} and transmission S_{21} coefficients are shown in Fig. 4. A notable difference between the expected stopband filter response of both guides is observed and proved by experiment. It verifies the condition that $R^{+\Omega} \neq L^{+\Omega}$. This effect disappears when the slab in one of the positions is reversed. Hence, we have that $R^{+\Omega} = L^{-\Omega}$. It is also expected in the situation when the magnetoelectric coupling in Ω medium vanishes i.e., the slab stands dielectric. Having accurate values of calculated and measured scattering parameters, we can make an attempt to find the proper values of the Ω medium used in our experiment. For extraction of the Ω material parameters, we use an optimization procedure based on the modified Marquardt–Levenberg algorithm. The problem is stated as follows.

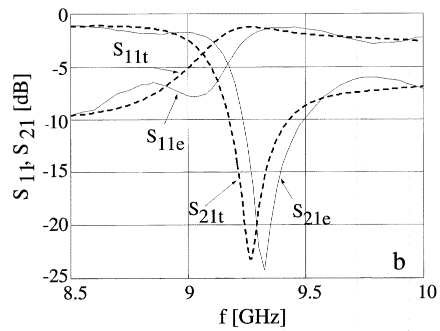
$$\min_{x \in \mathbf{R}} \tfrac{1}{2} \sum_{i=1}^{m} f_i(\underline{x})^2 \qquad (3)$$

where $\underline{x} = [\varepsilon'_y, \varepsilon''_y, \mu'_z, \mu''_z, \kappa', \kappa'']^T$ is a search vector of the complex dispersive material parameters. The remaining values $\varepsilon_x = \varepsilon_z$ and $\mu_x = \mu_y$ are assumed to be real and equal to host medium (duroid) parameters ε, μ. The goal of the optimization procedure is the minimization of (3) with $f_i(x) = S_m - S_t$, where S_m and S_t are magnitudes of scattering coefficients (S_{11}, S_{21}) measured and calculated at the fixed frequency for $M = 13$ positions of the slab in the guide. Hence, $2 * M = 26$ goal functions define (3).

As shown in Fig. 4, the simulated results calculated with the Engheta and Saadaun model medium parameters agree well

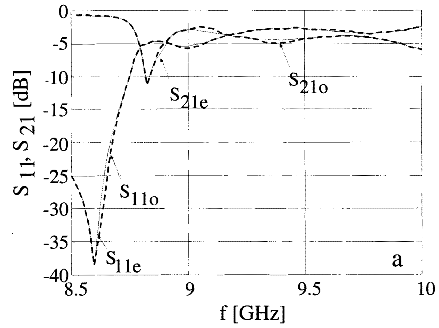


(a)

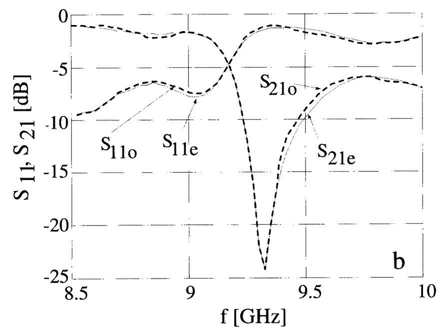


(b)

Fig. 4. Theoretical (dashed line) and experimental (solid line) characterizations of S_{11} and S_{21} of waveguide with Ω slab placed at (a) $W = 2$ mm and (b) $W = 10.86$ mm. Ω medium parameters from the Engheta and Saadaun model (see Fig. 5).



(a)



(b)

Fig. 6. Theoretical (dashed line) and experimental (solid line) characterizations of S_{11} and S_{21} of waveguide with Ω slab placed at (a) $W = 2$ mm and (b) $W = 10.86$ mm. Ω medium parameters after the Marquardt–Levenberg optimization procedure (see Fig. 5).

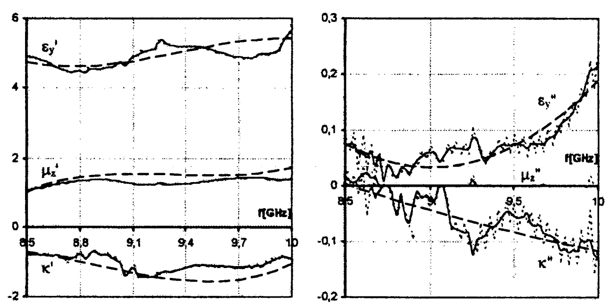


Fig. 5. The Ω material parameters with added noise characteristics (dashed line—the parameters calculated using the Engheta and Saadaun model; solid line—the optimization results) $\kappa = \eta_o * \Omega$ where η_o—intrinsic impedance of the free space.

with the measured ones. Therefore, the quantities of the parameters performed in Fig. 5 have been used to form the vector of starting data x.

We expect, then, that the output will be close to the global minimum of (3), giving a unique solution. For a stability check, noisy scattering parameters were generated with their mean values and different standard deviation $\delta < 5\%$. Only altered scattering parameters that fulfill energy conservation were considered. The extracted mean values of medium parameters with their deviations are plotted in Fig. 6. For these quantities (see Fig. 6), the agreement between theoretical and measured S-characteristics of the slab in the majority investigated positions is better then 3% and it is limited by 6% for other ones.

IV. Conclusion

We have examined a rectangular guide with a slab of pseudochiral Ω medium. Several unique and notable features associated with the considered structure were presented. The numerically and experimentally obtained differences between the scattering characteristics of the guides where Ω slab takes on symmetrical position have confirmed the previously predicted [6] field displacement effect. The opposite direction of the field displacement is observed when the Ω slab in the guide is reversed. The development of optimization Marquardt–Levenberg method using measured and calculated S-matrix parameters of Ω guide allowed extracting constitutive parameters of the Ω material.

References

[1] M. M. Saadoun and N. Engheta, "A reciprocal phase shifter using novel pseudochiral or Ω-medium," *Microwave Opt. Technol. Lett.*, vol. 5, pp. 184–188, 1992.

[2] M. M. Saadoun, "The pseudochiral Ω-medium: Theory and potential applications," Dr. dissertation, University of Pennsylvania, Sept. 1992.

[3] S. Tretyakov and A. Sochava, "Eigenwaves in uniaxial chiral omega media," *Microwave Opt. Technol. Lett.*, vol. 6, no. 12, pp. 701–705, Sept. 1993.

[4] J. Mazur and D. Pietrzak, "Field displacement phenomenon in a rectangular waveguide containing a thin plate of an Ω medium," *IEEE Microwave Guided Wave Lett.*, vol. 6, pp. 34–36, Jan. 1996.

[5] M. Norgren and S. He, "Reconstruction of the constitutive parameters for an Ω material in a rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 1315–1321, June 1995.

[6] J. Mazur and J. Michalski, "Pseudochiral omega medium in rectangular waveguides," in *7th Int. Conf. Complex Media*, 1998.